

Title: Points of Concurrency in a Triangle

Brief Overview:

This unit allows the student to discover the points of concurrency of a triangle using the TI-92 and Cabri. The student is guided to form conjectures about the properties of these points.

Links to Standards:

- **Mathematics as Problem Solving**
Students will demonstrate their ability to solve mathematical problems by correctly using the TI-92/Cabri as a problem solving tool.
- **Mathematics as Communication**
Students will summarize their findings with written conjectures in paragraph form.
- **Mathematics as Reasoning**
Students will demonstrate their ability to reason by forming conjectures about the relationship of each point of concurrency with the triangle.
- **Mathematical Connections**
Students will make a connection between their knowledge of circles and the points of concurrency in a triangle.

Grade/Level:

Grades 8-12

Duration/Length:

Two 50-minute periods or one block

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- TI-92/Cabri
- Classifications of triangles
- Definitions of perpendicular bisector, angle bisector, medians, altitudes
- Reading Geometric notation

Objectives:

Students will:

- discover the properties of the four points of concurrency for triangles.
- be able to construct circumscribed and inscribed circles.

Materials/Resources/Printed Materials:

- TI-92 calculator or Cabri software for each student
- The attached student directions

Development/Procedures:

These lessons are set up so that the student discovers the appropriate properties. Definitions are included or will be developed on the worksheet. Calculator hints for some steps follow this paragraph.

Perpendicular bisectors:

- Basic method for construction of a right triangle (Step 1).
Construct a horizontal segment CA (F2,5), labeling the points as you go (↑,letter).
Construct a line perpendicular to the segment at C (F4,1).
Put a point on the line (F2,1) and label this point B (↑,B).
Hide the perpendicular line (F7,1).
Draw the segments BC and BA (F2,5).
(This procedure is necessary to guarantee the triangle is right.)
- In Step 3: If the circumcenter is not on the screen, choose 2nd, grab hand and move the diagram on the screen. Show page (F8,A) will give an overview.
- In Step 4: When measuring segments, as soon as the measure is done, before enter is pressed, type the letters of the segment measured and an =. Have students drag measures to the side. These measures will be needed and will change as the triangle is transformed.
- In Step 5: Redefining point B is necessary to allow the point to be dragged. This eliminates the need to re-measure with each triangle.
- In Step 7: You may choose to adjust the precision of the calculator (F8,format) if the measures are not the same.

Angle bisectors:

- Dragging the vertex angle will not work in these examples and a separate triangle must be constructed for each type of triangle (F3,3).
- In Step 4: The line is needed to measure the distance from G to the side of the triangle.

Performance Assessment:

Teachers will monitor student performance during the work session(s) and evaluate the resulting conjectures.

Extension/Follow Up:

- Investigate isosceles and equilateral triangles.
- Investigate the orthocenter and centroid.
- Euler's Line and the Nine-Point Circle are possible extensions after the orthocenter and centroid are done.

Authors:

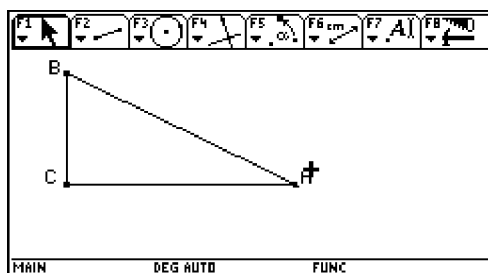
Beverly P. Boschert
Stuart High School
Fairfax County, VA

Ann W. Sparks
West Potomac High School
Fairfax County, VA

Two Points of Concurrency in a Triangle

Perpendicular bisectors:

1. Construct a right triangle, following your teacher's directions, and label the vertices A, B, C.
2. Construct the perpendicular bisectors of the 3 sides. (F4,4)
3. Put a point at the intersection of the 3 perpendicular bisectors and label it G. (F2,3), (F7,4)



When three or more lines intersect in one point, the point is called a **point of concurrency**.

4. Measure AG, BG, and CG and record your results in the table below. (F6,1)

| | Right Scalene | Acute scalene | Obtuse scalene |
|----|---------------|---------------|----------------|
| AG | | | |
| BG | | | |
| CG | | | |

5. Redefine point B as a point (F4,B,1). Grab and drag B to form an acute scalene triangle. Record the new measures above.
6. Grab and drag the same vertex in the opposite direction to create an obtuse triangle. Record the new measures above.
7. What do you notice about AG, BG, and CG? _____

Write a formal statement for any triangle that describes the relationship of this point of concurrency to the vertices.

Form a circle using G as the center (F3,1). Extend the circle until you think it is the circumscribed circle for this triangle. Write the definition of a circumscribed circle.

What constitutes the radius for this circle? _____
This point of concurrency is called the **circumcenter**.

Angle bisectors:

1. Construct an acute scalene triangle and label the vertices A, B, C. (F3,3)
2. Construct the angle bisector of each angle. (F4,5)
3. Put a point at the intersection of the 3 angle bisectors and label it G. (F2,3), (F7,4)
4. Overlay a line on each side of the triangle. (F2,4)
5. Measure the distance from G to the line on each side of the triangle. (F6,1)
6. Record the measures in the table below.

| Distance from G to | Acute Scalene | Obtuse Scalene | Right scalene |
|--------------------|---------------|----------------|---------------|
| AB | | | |
| AC | | | |
| BC | | | |

7. Construct an obtuse scalene triangle and complete the measures as above. Record the new measures above.
8. Repeat with a right scalene triangle.
9. What do you notice about the distances? _____

Write a formal statement for any triangle that describes the relationship of this point of concurrency to the sides.

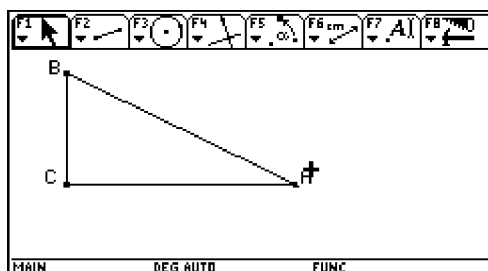
Form a circle using G as the center (F3,1). Extend the circle until you think it is the inscribed circle for this triangle. Write the definition of an inscribed circle.

This point of concurrency is called the **incenter**.

Two Points of Concurrency in a Triangle - Answer Key

Perpendicular bisectors:

1. Construct a right triangle and label the vertices A,B,C.
2. Construct the perpendicular bisectors of the 3 sides. (F4,4)
3. Put a point at the intersection of the 3 perpendicular bisectors and label it G.



When three or more lines intersect in one point, the point is called a **point of concurrency**.

4. Measure AG, BG, and CG and record your results in the table below.

| | Right Scalene | Acute scalene | Obtuse scalene |
|----|-------------------|---------------|----------------|
| AG | answers will vary | | |
| BG | | | |
| CG | | | |

5. Redefine point B as a point (F4,B,1). Grab and drag B to form an acute scalene triangle. Record the new measures above.
6. Grab and drag the same vertex in the opposite direction to create an obtuse triangle. Record the new measures above.
7. What do you notice about AG, BG, and CG? The measures are the same.

Write a formal statement for any triangle that describes the relationship of this point of concurrency to the vertices.

The point of concurrency for the perpendicular bisectors of any triangle is equidistant from the vertices of the triangle.

Form a circle using G as the center (F3,1). Extend the circle until you think it is the circumscribed circle for this triangle. Write the definition of a circumscribed circle.

A circle is circumscribed about a triangle if each vertex of the triangle lies on the circle.

What constitutes the radius for this circle? The distance from the point of concurrency to any vertex of the triangle is the radius of the circumscribed circle.

This point of concurrency is called the **circumcenter**.

Angle bisectors:

1. Construct an acute scalene triangle and label the vertices A,B,C. (F3,3)
2. Construct the angle bisector of each angle. (F4,5)
3. Put a point at the intersection of the 3 angle bisectors and label it G.
4. Overlay a line on each side of the triangle. (F2,4)
5. Measure the distance from G to each side of the triangle. (F6,1)
6. Record the measures in the table below.

| Distance from G to | Acute Scalene | Obtuse Scalene | Right scalene |
|--------------------|---------------|----------------|---------------|
| AB | answers vary | | |
| AC | | | |
| BC | | | |

7. Construct an obtuse scalene triangle and complete the measures as above. Record the new measures above.
8. Repeat with a right scalene triangle.
9. What do you notice about the distances? The distances are equal.

Write a formal statement for any triangle that describes the relationship of this point of concurrency to the sides.

The point of concurrency for the angle bisectors of any triangle is equidistant from the sides of that triangle.

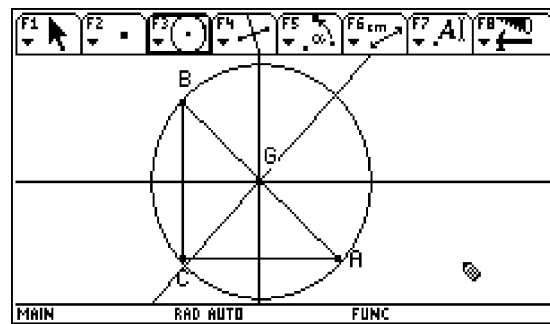
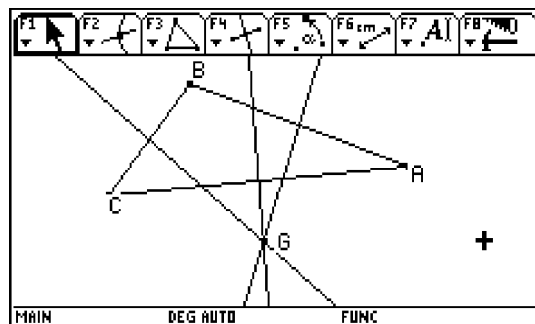
Form a circle using G as the center (F3,1). Extend the circle until you think it is the inscribed circle for this triangle. Write the definition of an inscribed circle.

A circle is inscribed in a triangle if the circle touches each side at exactly one point.

This point of concurrency is called the **incenter**.

Screen examples:

Perpendicular Bisectors:



Angle Bisectors:

